Effective field theory for the quantum electrodynamics of a graphene wire

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We study the low-energy quantum electrodynamics of electrons and holes in a thin graphene wire. We develop an effective field theory (EFT) based on an expansion in p/p_T, where p_T is the typical momentum of electrons and holes in the transverse direction, while p are the momenta in the longitudinal direction. We show that, to the lowest order in (p/p_T), our EFT theory is formally equivalent to the exactly solvable Schwinger model. By exploiting such an analogy, we find that the ground state of the quantum wire contains a condensate of electron-hole pairs. The excitation spectrum is saturated by electron-hole collective bound states, and we calculate the dispersion law of such modes. We also compute the dc conductivity per unit length at zero chemical potential and find $g_s\frac{e^2}{h}$, where $g_s=4$ is the degeneracy factor.

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I. INTRODUCTION

Monolayer graphene is the newest two-dimensional electronic system which has been fabricated in the last years. This system has attracted an enormous interest both from the experimental and theoretical sides since its electrons behave as Dirac fermions and show an unusual sequence of Landau levels yielding to an anomalous quantum-Hall effect.[1,2] These features of graphene are based on a tight-binding model calculation of its energy bands,[3–5] which shows a nearly linear dispersion of the energy bands close to the K point. Information on the energy dispersion of the Dirac cones has been achieved by means of Raman spectroscopy.[6–9]

Another important property of monolayer graphene is the possibility of cutting out samples of desired form and size. Of particular interest among these are quantum wires of graphene, which have been previously considered in several approaches.[10–12] In these studies, the properties of the quantum wires have been investigated theoretically for different types of confinement and of boundary conditions at the edge of the strip, neglecting the interaction among the carriers.

In this work, we study the consequences of the electromagnetic interactions between electrons and holes, inside a wire made from a single layer of two-dimensional graphene, at zero chemical potential. We show that the combined effects of the interaction and of the low dimensionality of the system drastically change the structure of the ground state and of the spectrum of excitations, which are typical of the two-dimensional graphene. In fact, the vacuum develops a condensate of electron-hole pairs and the spectrum of excitations turns out to be saturated by bosonic particle-hole collective modes.

Let v be the band velocity, a be the lattice spacing, and L and λ be the length and the width of the wire, respectively. In graphene, $v\simeq 10^6$ m s$^{-1}\simeq 1/300c$ and the lattice spacing $a\simeq 0.25$ nm. We consider a system for which $a\ll\lambda, L$ so one can adopt a continuous formulation. In addition, we assume $\lambda\ll L$ so that the energy associated to longitudinal momenta $E_L=\hbar v/L$ decouples from the energy associated to transverse momenta, $E_T=\hbar v/\lambda$.

Our goal is to exploit such a separation of scales to build an effective field theory (EFT), in which the explicit low-energy degrees of freedom are the electrons and holes which propagate in the longitudinal direction and interact by exchanging quanta of the electromagnetic field. We shall show that the interactions drastically change the properties of the wire: the spectrum gets saturated by electron-hole collective bound states. The electric conductivity turns out to be $\frac{e^2}{h}$ in units of the degeneracy factor $g_s=4$, i.e., the same result of Landauer's formula for the conductivity of a classical one-dimensional wire.[13]

The paper is organized as follows. In Sec. II we introduce the model Hamiltonian describing the motion of electron and holes without taking into account the electrostatic interactions between charge carriers. In Sec. III we briefly review the main ideas of EFT and construct the lowest-order Lagrangian, in which the effect of Coulomb interactions is systematically taken into account. In Sec. IV we show that, under suitable approximations, such a Lagrangian reduces to the Schwinger model. In Sec. V we shall review the exact solution of such a theory. The implications of these results on graphene physics will be presented in Sec. VI. Conclusions and perspective developments are summarized in Sec. VII.

II. THEORY FOR GRAPHENE WIRES WITHOUT COULOMB INTERACTIONS

We begin by considering the free quantum motion near the K point of the electrons in the conductance band and holes in the valence band of graphene. The corresponding second-quantized Hamiltonian leading to the desired single-particle spectrum

$$\omega(k) = \pm \hbar v|k| \tag{1}$$

is of course

$$H_0 = v\hbar \int dx \psi^\dagger(t,x)(-i\sigma_1\partial_x)\psi(t,x), \tag{2}$$

where $\psi(t,x)$ is the fermion-field operator. In the following, we shall work in the "natural units" of this problem, in which $\hbar=v=1$. Note that in such units the speed of light is $c=1/\beta\simeq 300$.

In order to exploit the formal analogy with the relativistic Dirac theory, it is convenient to introduce position contravariant vectors,

$$\tilde{x}^{\mu} = (vt, x) = (t, x), \tag{3}$$

momentum contravariant vectors,

$$\tilde{p}^{\mu} = (E/v, p) = (E, p), \tag{4}$$

and the metric tensor as $g_{\mu\nu}=\mathrm{diag}[1,-1]$. In addition, let γ^{μ} and γ^{S} be the 2×2 matrices obeying the usual Dirac algebra

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}, \tag{5}$$

$$\{\gamma^{S}, \gamma^{\mu}\} = 0, \tag{6}$$

and $(\gamma^{S})^2=1$. For example, one may choose a representation in which

$$\gamma^0 = \sigma_3,$$

$$\gamma^1 = i\sigma_2,$$

$$\gamma^S = \gamma^0\gamma^1 = \sigma_1. \tag{7}$$

Note that with this choice one has $\gamma^{\mu}\gamma^{S}=-\epsilon^{\mu\nu}\gamma_{\nu}$.

Using such a set of definitions, the action associated to the free "Dirac" Hamiltonian (2) can be cast in the familiar form

$$S_0 = \int d^2\tilde{x}\bar{\psi}i\tilde{\partial}\psi. \tag{8}$$

The purpose of this work is to study how the properties of the free Dirac particles described by Hamiltonian (2) are modified once electromagnetic interactions are systematically introduced. The construction of the Lagrangian for the theory with electromagnetic interaction will be presented in the next section using the framework of EFT.

III. EFFECTIVE FIELD THEORIES

Effective field theories are systematic low-energy approximations of more microscopic theories—for a pedagogic introduction see, e.g., Ref. 14. The main idea underlying the EFT formalism is the familiar observation that the low-energy processes are insensitive to the *details* of the ultraviolet physics. For example, the long-wavelength classical radiation generated by complicated current source $\mathbf{J}(\mathbf{r},t)$ of size d can be replaced by a sum of pointlike multipole currents $E1, M1, \ldots$. The usefulness of such an expansion is that, for wavelengths $\lambda \gg d$, usually only few multipoles are usually needed for sufficient accuracy. The gain in going from the fundamental theory to its low-energy EFT resides in the fact that the latter is in general much simpler as it describes the dynamics of fewer degrees of freedom. Whenever an EFT can be built, all predictions for low-energy observables of the underlying microscopic theory can be reproduced to a desired level of accuracy, truncating the expansion at a finite order. However, the price to pay for such a simplification is that of having to specify a finite number of unknown parameters. These have to be determined from experiment or from microscopic calculations in the underlying (more) fundamental theory.

The transformation from a microscopic theory to a simpler EFT valid for low-momentum processes is achieved in two steps. The first step consists in introducing a hard cutoff of order the momentum at which the physics we are not interested in describing becomes important. Only momenta lower than such a cutoff are retained in calculations. The second step consists in writing the Lagrangian or the Hamiltonian for the low-energy degrees of freedom. According to Weinberg theorem,[15] in quantum field theory this task can be achieved starting from the most general Lagrangian, compatible with the symmetry properties of the underlying fundamental theory and with the fundamental principles of quantum field theory,

$$\mathcal{L} = \sum_{i=1}^{\infty} c_i(\Lambda)\hat{O}_i. \tag{9}$$

Note that the effective Lagrangian $\mathcal{L}$ contains infinitely many operators O_i, along with an infinite number of corresponding unknown effective coefficients c_i. Hence, expression (9) has no predictive power, thus does not yet represent a physical theory. The usefulness of the EFT scheme resides in the possibility of establishing a power-counting scheme, i.e., a prescription to retain only a finite number of terms in the effective Lagrangian [Eq. (9)]. This possibility is based on the observation that the contribution of the operators with higher and higher dimensionality in Eq. (9) to any Green's function with external momenta of order p is suppressed by powers of the ratio of momentum over the cutoff, i.e., p/Λ. Hence, predictions to an arbitrary level of accuracy can be obtained by retaining only a *finite* number of lowest-dimensional operators in Eq. (9). The corresponding effective coefficients have to be determined from microscopic calculations in the underlying fundamental theory or by performing a finite number of experiments.

As an example, let us consider the EFT for the ordinary QED in vacuum, for momenta much below a cutoff scale Λ, chosen above the electron mass. In $3+1$ dimension and natural units ($\hbar=c=1$) the electron field and the photon field have mass dimension 3/2 and 1, respectively, while the electric charge is dimensionless. Hence, the lowest-dimensional operators which appear in the most general Lagrangian compatible with Lorentz and gauge symmetry are,

$$\begin{aligned}\mathcal{L}^{(\Lambda)} = {} & \bar{\psi}[i\partial\!\!\!/ - e(\Lambda)A\cdot\gamma - m(\Lambda)]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & + e(\Lambda)c_1(\Lambda)\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi + e(\Lambda)c_2(\Lambda)\bar{\psi}i\partial_{\mu}F^{\mu\nu}\gamma_{\nu}\psi \\ & + d_2(\bar{\psi}\gamma_{\mu}\psi)^2 + \cdots .\end{aligned} \tag{10}$$

Some comments on this expression are in order. First of all, we note that the second and third lines display terms which are not present in the original QED Lagrangian. The role of such effective local interactions is to mimic the ultraviolet physics above the cutoff Λ. Note also that the operators in different lines have different mass dimensions: the operators in the first line have dimension four, that in the second line

has dimension five, and those in the last line have dimension six. Since in natural units the action must be dimensionless, each term in the Lagrangian must have mass dimension equal to the number of space-time dimensions. This condition fixes the mass dimension of the effective coefficients to $[c_1]=-1$, $[c_2]=-2$, and $[d_2]=-2$.

A simple dimensional analysis implies that if the external momenta $p's$ are small compared to the cutoff then the contribution to the Green's functions of the terms in the second and third lines of Eq. (10) are suppressed by increasing powers of (p/Λ) with respect to the contribution of the terms in the first line. Hence, if the cutoff is sent to infinity, only the first line provides finite contributions. For this reason, in the language of effective field theory, the operators with mass dimension larger than the number of space-time dimensions are called *irrelevant*.

Finally, we observe also that effective field theories are in general nonrenormalizable. However, that does not lead to problems because the cutoff Λ is always kept finite. The effective field theory [Eq. (10)] retains predictive power at the price of computing the coefficients $c_1(\Lambda)$, $c_2(\Lambda)$, and $d_2(\Lambda)$ microscopically from QED or to fix them from experimental measurements.

Let us now carry out the same program in our specific case, in which the cutoff scale is provided by the transverse momentum in the wire, $\Lambda=p_T$. We are interested in an effective field theory which describes only the dynamics of the low-energy degrees of freedom inside the wire, i.e., the electrons, holes, and photons propagating along the longitudinal direction. On the other hand, we are not interested in describing the dynamics of the on-shell photons, which are radiated away from the wire. Hence, it is sufficient to consider an effective field theory in $1+1$ dimensions, in which a *dynamical* two-component photon effective field is introduced to mediate the electron-hole interaction, inside the wire. In addition, the electrons and holes are allowed to interact with an *external* electromagnetic field, which is therefore defined in $3+1$ dimensions.

The starting point to construct an effective Lagrangian which satisfies these requirements is to introduce a gauge-invariant coupling of photons and fermions,

$$\tilde{D}_\mu = \tilde{\partial}_\mu + ie\ \tilde{a}_\mu + ie\tilde{A}^{\text{ext}}_\mu, \tag{11}$$

where we have used the following definitions:

$$\tilde{a}_0 = a_0, \quad \tilde{A}^{\text{ext}}_0 = A^{\text{ext}}_0, \tag{12}$$

$$\tilde{a}_1 = \beta a_1, \quad \tilde{A}^{\text{ext}}_1 = \beta A^{\text{ext}}_1. \tag{13}$$

Here $\tilde{a}_\mu$ is the quantized dynamical field which describes the photons inside the wire, while A^{ext}_μ is a classical external field. Notice that, in $1+1$ dimensions, the fermion field has mass dimension ½, while the a_μ and A_μ field have mass dimension one. The lowest-dimensional gauge-invariant and "Lorentz"-invariant operators are

$$\bar{\psi} i \tilde{\not{D}} \psi, \quad m(\Lambda)\bar{\psi}\psi, \quad (\bar{\psi} O \psi)^2, \quad (O = 1, i\gamma_s, \tilde{\gamma}_\mu, \tilde{\gamma}_\mu\gamma_S),$$

$$\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi,$$

$$f_{\mu\nu}f^{\mu\nu}, \quad \bar{\psi} i \tilde{\partial}_\mu F^{\mu\nu}\gamma_\nu \psi. \tag{14}$$

The first, second, and third lines contain operators of dimensions two, three, and four, respectively. Some of such terms can be rejected based on physical considerations. First of all, we want our theory to reproduce the free theory results, in the limit in which the electromagnetic coupling is set to zero. In order to ensure such a condition, we must not include the mass term, which would spoil the linearity of the fermion dispersion relation, near the K point.

The term $\bar{\psi}\sigma_{\mu\nu}F^{\mu\nu}\psi$ describes in general the interaction of the electromagnetic field with the magnetic moment of the fermion field. However, one should keep in mind that the Pauli matrices in the Dirac-type theory for graphene do not have the real physical interpretation of spin matrices. Clearly, an interaction between magnetic field and pseudospin would not be physical.

The term $\bar{\psi} i \tilde{\partial}_\mu F^{\mu\nu}\gamma_\nu\psi$ can be eliminated using the equation of motion for the a_μ field generated by the terms which are present at this order. Hence, its only effect is that of rescaling the existing effective coefficients.

The contact terms $(\bar{\psi} O \psi)^2$ deserve a particular attention. In principle, these terms appear already at the lowest order in our effective Lagrangian and should be kept into account. Their role is to mimic the ultraviolet physics which sets in when electrons and holes interact at a distance of the order of the inverse cutoff, i.e., of the transverse size of the wire, $\simeq 1/p_T$. The inclusion of such terms could, in principle, be dealt in the context of the massless Thirring model.[16] However, in the limit of extremely small electron density we are considering, the correlations brought in by the contact interactions are certainly small compared to the long-ranged Coulombic interaction, provided by the $\bar{\psi} i \tilde{\not{D}} \psi$ and $f_{\mu\nu}f^{\mu\nu}$ terms. Hence, we chose to neglect the effects of the contact terms in our first work. Finally, we observe that a finite electron density would give raise to a term $\mu\bar{\psi}\gamma_0\psi$, which would appear at the leading order, as expected.

Strictly speaking, some of the approximations made above could spoil the power-counting scheme of our effective field theory. This means that our results may retain some leading-order dependence on the specific choice of the cutoff. However, in our specific case, this does not represent a problem. In fact, unlike the example of low-energy QED, the choice of the cutoff is not arbitrary but it is determined by the geometry of the wire, i.e., by its transverse size.

In conclusion, our lowest-order effective action for the internal quantum electrodynamics of the wire reads

$$S \equiv \int d^2\tilde{x} \left[\bar{\psi} i \tilde{\not{D}} \psi - \frac{c(\Lambda)}{4} f_{\mu\nu}f^{\mu\nu} \right], \tag{15}$$

where

$$f_{\mu\nu} = \tilde{\partial}_\mu \tilde{a}_\nu - \tilde{\partial}_\nu \tilde{a}_\mu = \partial_\mu a_\nu - \partial_\nu a_\mu.$$

IV. MAPPING THE ELECTRODYNAMICS OF THE WIRE ONTO THE SCHWINGER MODEL

The present work is driven by the observation that effective action [Eq. (15)] can be formally mapped onto QED with massless fermions in 1+1 dimensions, i.e., in the celebrated chiral *Schwinger model*.[17] This is an exactly solvable model which exhibits an extremely rich nonperturbative dynamics—for a detailed discussion see, e.g., Refs. 17 and 18 and references therein. To establish such a connection it is sufficient to reabsorb the effective coefficient $c(\Lambda)$ in the definition of the photon field by setting $\sqrt{c(\Lambda)}\tilde{a}_\mu \to \tilde{a}_\mu$. Once this has been done, a factor $1/\sqrt{c(\Lambda)}$ appears in the electromagnetic coupling operator between the fermions and the rescaled photon field. In conclusion, at the leading order in the expansion parameter $(p\lambda)$, the quantum electrodynamics of electrons and holes inside the wire is described by the effective action

$$S = \int d^2\tilde{x}\left[\bar{\psi} i \tilde{\not{D}} \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu}\right], \quad \mu,\nu = 0,1, \tag{16}$$

where the covariant derivative is defined as

$$\tilde{D}_\mu = \tilde{\partial}_\mu + ig\ \tilde{a}_\mu + ie\tilde{A}^{\text{ext}}_\mu, \tag{17}$$

with $g = \frac{e}{\sqrt{c(\Lambda)}}$.

Equation (16) defines the action of the Schwinger model. We stress the fact that factor $1/\sqrt{c(\Lambda)}$ appears only in the coupling of the fermions with the *quantized* and rescaled electromagnetic field $\tilde{a}_\mu$ and not in the coupling with the classical external field A^{ext}_μ. We also emphasize the fact that such a coefficient is not predicted in our approach. It has to be determined either from experiments or from microscopic calculations.

V. EXACT SOLUTION OF THE SCHWINGER MODEL

A remarkable feature of massless QED in 1+1 dimensions is that the variation in the action generated by the two independent rotations of the fermion fields

$$\psi(x) \to e^{i\chi(x)}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\chi(x)}, \tag{18}$$

$$\psi(x) \to e^{i\gamma^S\Phi(x)}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{i\gamma^S\Phi(x)} \tag{19}$$

can be reabsorbed by two local gauge transformations,

$$\tilde{a}_\mu \to \tilde{a}_\mu - \frac{1}{g}\tilde{\partial}_\mu\chi, \tag{20}$$

$$\tilde{a}_\mu \to \tilde{a}_\mu + \frac{1}{g}\epsilon_{\mu\nu}\tilde{\partial}^\nu\Phi. \tag{21}$$

The dynamical consequences of this fact become evident once one parametrizes the photon field degrees of freedom as

$$\tilde{a}_\mu = \frac{1}{g}(\tilde{\partial}_\mu\chi - \epsilon_{\mu\nu}\tilde{\partial}^\nu\phi) \tag{22}$$

and re-expresses the path integral in terms of the fermion fields $\psi, \bar{\psi}$ and of the χ and ϕ fields. The χ field is pure gauge, hence it has a vanishing field-strength tensor. In addition, the fermionic measure $\mathcal{D}\psi\mathcal{D}\bar{\psi}$ is left invariant by the gauge rotation [Eq. (18)]. Hence, the χ field is unphysical and can be completely eliminated from the path integral. On the other hand, the ϕ field contributes to the field strength tensor, through the term $2\frac{1}{g^2}\tilde{\partial}^\mu\phi\tilde{\partial}^2\tilde{\partial}_\mu\phi$. In addition, an anomalous term appears in the action, as a consequence of the fact that the chiral rotation [Eq. (19)] does not leave the functional measure of the fermion-fields invariant,

$$\mathcal{D}\psi\mathcal{D}\bar{\psi} \to \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{J}^{-2}, \tag{23}$$

where

$$\mathcal{J}^{-2} = \exp\left[-2\int d^2\tilde{x}\frac{1}{g^2}\tilde{\partial}^\mu\phi m_\phi^2\tilde{\partial}_\mu\phi\right] \tag{24}$$

is the functional Jacobian determinant of the chiral transformation [Eq. (19)] and $m_\phi = g/\sqrt{\pi}$ is the so-called Schwinger "mass."

As a result, the path integral of the leading-order effective action reads

$$Z = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\phi\ \exp[iS_{\text{eff}}(\psi,\bar{\psi},\phi)], \tag{25}$$

where the effective action S_{eff} is defined as

$$S_{\text{eff}}[\psi,\bar{\psi},\phi] = \int d^2\tilde{x}\bar{\psi} i\tilde{\not{\partial}}\psi - \frac{1}{2}\frac{1}{g^2}\tilde{\partial}^\mu\phi(\tilde{\partial}^2 - m_\phi^2)\tilde{\partial}_\mu\phi. \tag{26}$$

The effective action [Eq. (26)] is quadratic in the both the fermion and boson fields. Thus, the path integral defining the corresponding quantum theory is Gaussian so arbitrary n-point Green's functions can be evaluated exactly. All technical details of such calculations can be found in the original papers,[17,18] and in standard quantum-field-theory books, such as, e.g., Refs. 16 and 19 and will not be repeated here.

Let us begin by discuss the vacuum structure. The spectrum of scalar states is related to the poles of the two-point Green's function,

$$G(q) = \int d^2\tilde{x}e^{i\tilde{q}\cdot\tilde{x}}\langle\Omega|T[\bar{\psi}(\tilde{x})\psi(\tilde{x})\psi(0)\psi(0)]|\Omega\rangle. \tag{27}$$

After performing the analytic continuation to the Euclidean space one finds (see, e.g., Ref. 16)

$$\langle 0|T[\bar{\psi}(\tilde{x})\psi(\tilde{x})\psi(\tilde{0})\psi(0)]|0\rangle = \frac{K}{2\pi^2\tilde{x}^2}e^{-4\pi[-\Delta(m_\phi,\tilde{x})+\Delta(0,\tilde{x})]}, \tag{28}$$

where $\Delta(m_\phi,\tilde{x})$ is the Fourier transform of the ϕ-field propagator,

$$\Delta(m_\phi,\tilde{x}) = \int \frac{d^2\tilde{p}}{4\pi^2}\frac{1}{\tilde{p}^2 + m_\phi^2}e^{i\tilde{p}\cdot\tilde{x}} \tag{29}$$

and

$$K = e^{-4\pi[\Delta(m_\phi,0)-\Delta(0,0)]}. \tag{30}$$

It is immediate to verify that the point-to-point Green's function (28) does not vanish at large Euclidean distances. This implies that the vacuum has a nontrivial structure: it contains a finite density of fermion-antifermion pairs

$$\langle\Omega|\bar{\psi}\psi|\Omega\rangle = -\frac{g}{2\pi\sqrt{\pi}}\exp(\gamma), \tag{31}$$

where γ is the Euler constant. Note that, in this expression, we have dropped space-time labels since the ground state is static and invariant under translations along the longitudinal directions.

Another remarkable feature of the Schwinger model is that all the zero-mass singularities of Green's function (27) exactly cancel out. The only singularity are free bosonic states located at

$$\tilde{q}^2 = n^2 m_\phi^2, \quad n = 1,2,3,\ldots. \tag{32}$$

The physical interpretation of this fact is that fermionic excitations are lifted from the spectrum and only bosonic collective states exist.

This result can be proven as follows. First, we bosonize the free fermion fields $\bar{\psi}$ and ψ, introducing a free massless boson field θ. The action becomes

$$S[\theta,\phi] = \int d^2\tilde{x}\frac{1}{2}(\tilde{\partial}_\mu\theta)^2 - \frac{1}{2}\frac{1}{g^2}\tilde{\partial}^\mu\phi(x)(\tilde{\partial}^2 - m_\phi^2)\tilde{\partial}_\mu\phi(x). \tag{33}$$

Upon performing a translation of the θ field,

$$\theta + \phi/\sqrt{\pi} \to \theta, \tag{34}$$

and integrating over ϕ, one obtains a quantum field theory defined by the action

$$S[\theta] = \int d^2\tilde{x}\frac{1}{2}(\tilde{\partial}_\mu\theta)^2 - \frac{1}{2}m_\phi^2\theta^2, \tag{35}$$

which corresponds to a free theory of scalar bosons.

VI. PHYSICAL PROPERTIES OF THE GRAPHENE WIRE WITH COULOMB INTERACTIONS

Based on the mapping between the Schwinger model and our EFT for the electrodynamics of the graphene wire, we can use the results listed in the previous section to obtain nontrivial predictions for the physical properties the system we are considering. In the following, we list some most important predictions for the physical properties of the graphene wire at zero temperature and chemical potential.

A. Ground-state structure

The existence of a finite vacuum condensate [Eq. (31)] means that the ground state of the quantum wire does not correspond to a configuration in which there is no electron in the conductance band and no hole in the valence band. Instead, it contains a finite density of scalar electrons-hole pairs.

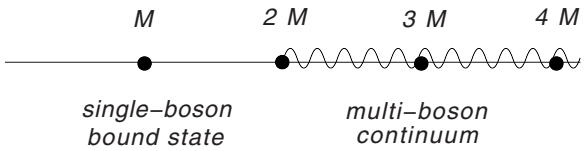

FIG. 1. Structure of the spectrum of collective excitations in the graphene wire.

B. Bosonization of the spectrum of excitations

In the previous section, we have seen that the spectrum of the Schwinger model does not contain single fermion excitations but only arbitrary number of free fermion-antifermion bound states (Schwinger bosons), of mass $M = g^2/\pi$, and spin zero. In the context of graphene theory, this feature implies that the spectrum of excitations of the wire starts with a single collective boson excitation with dispersion relation

$$\omega(k) = \pm\hbar v\sqrt{k^2 + \frac{4\alpha/\beta}{c(\Lambda)}} \tag{36}$$

and contains a continuous of multiboson excitations, starting at the two-boson threshold. Additional thresholds for multiboson excitations are located at nM, with $n=3,4,5,\ldots$ (see Fig. 1). In these formulas, we have restored the v, c, and $\hbar$ constants and $\alpha = \frac{e^2}{4\pi\hbar c} \simeq 1/137$ is the fine-structure constant.

The bosonization of the spectrum is a consequence of the fact that the theory is defined in 1+1 dimension and the fermions are massless. Indeed, it is observed also in the Luttinger liquids. However, an interesting feature of the graphene wire is that the bosonization occurs at vanishing chemical potential. Note that from the measurement of the gap $\Delta = 2\sqrt{\frac{\alpha/\beta}{c(\Lambda)}}$ it is, in principle, possible to determine the leading-order effective coefficient $c(\Lambda)$ of our EFT.

C. Conductivity of the graphene wire

The electric conductivity of the graphene wire is defined as

$$\Re[\sigma(\omega,q)] = \frac{1}{E^{\rm ext}(\omega,q)}\Re\int d^2\tilde{x}e^{i(\tilde{\omega}\tilde{x}_0 - \tilde{q}\tilde{x})}\langle j_1(\tilde{x})\rangle_{\tilde{A}^\mu}, \tag{37}$$

where $\tilde{A}^\mu(\tilde{x}) = [\Phi(\tilde{x}),0]$ the potential of a weak external electric field, $E^{\rm ext}$.

Applying the linear-response theory, one immediately finds

$$\int d^2\tilde{x}e^{i\tilde{q}\cdot\tilde{x}}\langle j^1(\tilde{x})\rangle_{\tilde{A}_0^\mu} = i[i\Pi^{10}(\tilde{q})]\tilde{A}_0^{\rm ext}(\tilde{q}), \tag{38}$$

where $\Pi^{\mu\nu}(\tilde{q})$ is the vacuum polarization tensor, defined as,

$$i\Pi^{\mu\nu}(\tilde{q}) = \int d^2\tilde{x}e^{i\tilde{q}\cdot\tilde{x}}\langle\Omega|T[j^\mu(\tilde{x})j^\nu(0)]|\Omega\rangle. \tag{39}$$

In the Schwinger model, this matrix elements can be computed exactly and reads

$$i\Pi^{\mu\nu}(\tilde{q}) = -\left(g^{\mu\nu} - \frac{\tilde{q}^{\mu}\tilde{q}^{\nu}}{\tilde{q}^2}\right)\frac{g^2}{\pi}. \tag{40}$$

This result can be used to readily obtain the conductivity of the wire. After restoring the appropriate powers of v and $\hbar$, we find our final results,

$$\Re\left[\frac{\sigma(\omega,q)}{L}\right] = \Im\frac{e^2}{\pi\hbar}\frac{\omega/v}{(\omega/v+q)(\omega/v-q)}, \tag{41}$$

and by taking the Fourier transform[20]

$$\Re\left[\frac{\sigma(\omega,x)}{L}\right] = \frac{e^2}{2\pi\hbar}\cos\left(\frac{\omega x}{v}\right). \tag{42}$$

Thus in the limit $\omega\to 0$, we get for the dc conductance G (defined as the $\omega\to 0$ limit of $\Re[\frac{\sigma(\omega,x)}{L}]$),

$$G = \frac{e^2}{h}. \tag{43}$$

This result must then be multiplied for the factor $g_s=4$ to account for the spin and sublattice degeneracy.

A few comments on this result are in order. First of all, we note that the conductance does not depend on the effective coefficient and therefore on the scale factor λ. Furthermore, we note that the current-current correlation function [Eq. (40)] is completely saturated by the pole corresponding to the Schwinger boson. This means that the correlation between the currents is mediated by the exchange of a massive composite bound state. The current induced by the external field is a purely quantum effect mediated by the chiral anomaly. Physically, it arises from the polarization of the electron and holes constituents inside the massive Schwinger bosons (see Fig. 2).

Finally, we observe that the result [Eq. (43)] for the conductance is in good agreement with a recent experimental observation of subband formation in graphene nanoribbon.[22]

VII. CONCLUSIONS

In this work, we have presented the first study of the effects of the electric interactions between electrons and holes inside a thin graphene wire. To this end, we have developed EFT based on an expansion in p/p_T, and we have shown that the lowest order of such a theory can be formally mapped onto the exactly solvable Schwinger model. Using such an equivalence, we have shown that the electromagnetic interaction between electron and holes in a quasi-one-dimensional wire dramatically affects its longitudinal dispersion law, already at zero chemical potential. The spectrum of the wire contains only collective particle-hole excitations with dispersion characterized by a finite gap. The dc conductance at zero chemical potential is given by $\frac{e^2}{h}$, in units of the degeneracy factor $g_s=4$.

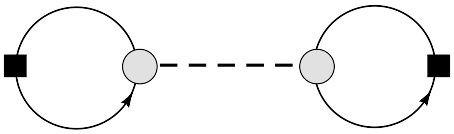

FIG. 2. Graphical representation of the current-current correlation function $\Pi_{\mu\nu}$ in a graphene wire. The black square represents the electromagnetic current operator $J^{\mu}=\bar{\psi}\gamma^{\mu}\psi$, the dashed line denotes the Schwinger boson propagator, while the gray circles represent the electron-hole wave function in the Schwinger boson state. The induced current is a consequence of quantum fluctuations of the Schwinger boson into electron-holes pairs. This figure was drawn using JAXODRAW (Ref. 21).

It should be stressed that the present model is expected to work only in an appropriate range of length and width. In particular, if the width of the wire is too narrow, the shape of the edge will significantly affect the electronic structure and change the Dirac equation in the continuum approximation. On the other hand, if the wire is too wide, the lowest-order approximation of the present EFT would become insufficient. Hence, it would be interesting to compare the predictions of our model with the results of lattice simulations for the electrodynamics two-dimensional graphene systems of different lengths and widths, using, e.g., the techniques developed in Ref. 23. On the one hand, this would provide a microscopic calculation of the unknown effective parameter $c(\Lambda)$. On the other hand, it would allow us to identify the region of transverse and longitudinal momenta, where the present EFT is applicable. Another possible development of the present work would be to investigate how the properties of the wire change as a function of the fermion chemical potential and of the temperature.

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[1] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, Nature (London) **438**, 197 (2005).
[2] Y. Zhang, Yan-Wen Tan, Horst L. Stormer, and Philip Kim, Nature (London) **438**, 201 (2005).
[3] F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).
[4] Y. Zheng and T. Ando, Phys. Rev. B **65**, 245420 (2002).
[5] N. M. Peres, A. H. Castro Neto, and F. Guinea, Phys. Rev. B **73**, 125411 (2006).
[6] A. C. Ferrari, J. C. Meyer, V. Scardaci, C. Casiraghi, M. Lazzeri, F. Mauri, S. Piscanec, D. Jiang, K. S. Novoselov, S. Roth, and A. K. Geim, Phys. Rev. Lett. **97**, 187401 (2006).
[7] D. Graf, F. Molitor, K. Ensslin, C. Stampfer, A. Jungen, C. Hierold, and L. Wirtz, Nano Lett. **7**, 238 (2007).
[8] J. Yan, Y. Zhang, P. Kim, and A. Pinczuk, Phys. Rev. Lett. **98**, 166802 (2007).
[9] S. Pisana, M. Lazzeri, C. Casiraghi, K. S. Novoselov, A. K. Geim, A. C. Ferrari, and F. Mauri, Nature Mater. **6**, 198 (2007).
[10] D. S. Novikov, Phys. Rev. Lett. **99**, 056802 (2007).
[11] P. G. Silvestrov and K. B. Efetov, Phys. Rev. Lett. **98**, 016802

(2007).
[12] J. Milton Pereira, Jr., F. M. Peeters, and P. Vasilopoulos, Phys. Rev. B **75**, 125433 (2007).
[13] R. Landauer, Philos. Mag. **21**, 863 (1970).
[14] G. P. Lepage, arXiv:nucl-th/9706029 (unpublished).
[15] S. Weinberg, *The Quantum Theory of Fields, Modern Applications* (Cambridge University Press, Cambridge, England, 2000), Vol. II.
[16] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, 2nd ed. (Oxford University Press, New York, 1993).
[17] J. Schwinger, Phys. Rev. **128**, 2425 (1962).
[18] A. Casher, J. Kogout, and L. Susskind, Phys. Rev. D **10**, 732 (1974).
[19] M. E. Pesking and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Perseus Book Group, Reading, MA, 1995).
[20] A. Kawabata, J. Phys. Soc. Jpn. **65**, 30 (1996).
[21] D. Binosi and L. Theul, Comput. Phys. Commun. **161**, 76 (2004).
[22] Y. M. Lin, V. Perebeinos, Z. Chen and P. Avouris, Phys. Rev. B **78**, 161409(R) (2008).
[23] J. E. Drut and T. A. Lahde, Phys. Rev. Lett. **102**, 026802 (2009); S. Hands and C. Strouthos, Phys. Rev. B **78**, 165423 (2008).